The SuperBall Lab

Objective

This goal of this tutorial lab is to introduce data analysis techniques by examining energy loss in super ball collisions.

Instructions

This laboratory does not have to be written up as a formal report. You only have to hand in a short report, generated by answering all the questions in the worksheet at the end of this document.

Before you come to this lab study the online notes on error analysis (Craft-Short.pdf) found at http://www.physics.queensu.ca/~phys106/labs.html

Introduction

As stated above, the goal of this lab is to examine energy loss in SuperBall collisions by dropping a ball and measuring the height to which it rebounds. Before the ball is released it has potential energy. As the ball drops, the potential energy is converted into kinetic energy. During the collision with the floor the ball is deformed and the kinetic energy is transformed into internal elastic energy within the ball. This process is then reversed and at its maximum altitude the ball once again has only potential energy. If the collision between the ball and the floor is perfectly elastic, and there is no air friction, the ball will lose no energy and it will rebound to the starting height.

Our goal is to measure the fraction of the energy that is lost in the collision. The formula for gravitational potential energy is $PE = mgh$, where $m$ is the mass of the object, $g$ is the acceleration due to gravity and $h$ is the height of the object relative to some reference point (in our case the floor). To find the energy loss would require us to determine the mass of the ball which is more work than we want to perform. Moreover, it introduces another measurement and therefore another source of error. Instead we will measure the elasticity of the ball, which is the fraction of its initial energy it retains after the collision, i.e.

$$Elasticity = \frac{mgh_i}{mgh_0} = \frac{h_i}{h_0} \quad (1)$$

where the subscript 0 denotes the initial height, and the subscript 1 denotes the height of the rebound. (Note that since the elasticity is a ratio, it is dimensionless.)
During this tutorial lab exercise you will use and compare three different data analysis techniques to calculate *Elasticity*:

1. **Single trial**
   - This is usually used when multiple trials are not possible.
   - *Error propagation* is used to determine uncertainty in the final calculated value from the uncertainty in the measurements.

2. **Statistical analysis**
   - This involves multiple trials of either identical situations or situations where one variable changes.
   - The desired value is the “mean” and the error is the standard deviation of the mean.

3. **Graphical analysis**
   - Here you vary one parameter, plot the data on a graph and do a *linear regression* to obtain the equation of the best fit line though your data. Note: If your experiment can only be repeated under identical conditions (i.e. repeating the same thing over and over) then graphical analysis is not possible.
   - The desired value is obtained from the slope of the fitted line or by combining the slope with other measured quantities. The error is obtained from the *standard error* on the slope, using the slope error in *error propagation* formulas if necessary.
   - When it is possible, **this method is always preferred for experimental analysis**. Why? Because in addition to giving you slope and error information it allows you to qualitatively evaluate your data by looking at the graph.

You will be given 2 balls and are expected to compare their elasticities. The procedures for determining uncertainty in measurements, collecting data and analysing data using the 3 different methods are given on the next few pages.
Measurement Error (Uncertainty)*

At some point in most experiments the measurement error should be determined.

- In the single trial case this information is used directly to calculate the uncertainty in the results (using error propagation).
- In statistical analysis it is actually not used at all!
- In graphical analysis the measurement error for the graphed values appear as error bars. These error bars tell you how closely your values match the expected behaviour and whether or not you need to repeat any measurements.

The measurement error we are interested in is the unavoidable error that we refer to as random error (this is also discussed in the online notes). There are two ways of estimating this random error:

1. Sometimes uncertainty depends only on your measuring device. In these cases the uncertainty is just half of the smallest division on your scale. For example, if you measure some clearly defined length using a ruler with millimetre graduations you might decide that you can measure the length to the nearest millimetre. Here the uncertainty would be half a millimetre.

2. However, there are often other errors in addition to those due to the limited resolution of our measurement device. In this case the best approach is to repeat a single measurement a few times (usually ~3) times. The uncertainty will be half of the range and the average is the actual value. If a length is measured four times and values range between 46mm and 48mm then we can estimate that the true values is 47 +/- 1mm (i.e. the uncertainty is half the range). This should not be confused with statistical analysis, where you repeat a measurement many times (>10) and get a mean and standard deviation of the mean.

These methods do not apply to systematic errors. Systematic errors are ones that affect all measurements the same way and so repeated measurements do not help. Systematic errors that result from poor experimental procedures should be identified and the procedure corrected to avoid them. For example, when you measure the height of the rebound, the height you read depends on the position of your head – this is called parallax. If you are always looking from above the ball measurements will always be too small. To correct this, look straight on at the ball.

* Note that the terms ‘error’ and ‘uncertainty’ mean the same thing in this case and we use them interchangeably.
Measurement error (uncertainty) for the bouncing superballs in this lab

For each ball determine the drop height uncertainty and rebound height uncertainty ($\Delta h_c$ and $\Delta h_r$).

- **Drop height uncertainty**: you might try having one partner hold the ball as if they were about to drop it and read the height of the ball. The other partner can then also read the height of the ball. Due to slightly different head positions (parallax) these values will likely be different. Do this several times and determine the range of variation.

- **Rebound height uncertainty**: To estimate the error in the rebound height you probably want to drop the ball several (3-4) times from a given height. Determine the range of rebound readings you get to get your measurement error as described on the previous page. What causes the readings to vary? (i.e. what are the sources of random error?)

**Study 1: Single trial**

First of all you should have determined your measurement error using the method described above. The elasticity of the ball can be measured by dropping the ball from a known drop height and recording the rebound height. Do this once – this is your single trial. Substituting the drop height and rebound height from this single trial into the equation for elasticity (1) gives an estimate of the elasticity of the ball. The error for elasticity is determined by using the measurement error for the drop and rebound heights in the error propagation formula.

**Procedure**

Drop each ball from a known height and record the height of the rebound. Calculate the elasticity of each ball based on a single drop. (Equation 1).

Using error propagation, determine the uncertainty in the calculated elasticities.
Study 2: Statistical analysis

A statistical estimate of the elasticity of each ball can be made by performing several trials (usually >10), calculating the elasticity for each trial, and taking the mean of these single trial values. The standard deviation of the mean is the uncertainty of this estimate.

Procedure

Note: For this analysis the balls could be dropped repeatedly from the same height. However, by using different heights we can use the same data in the next experiment involving graphical analysis.

Drop each ball from 10 to 12 different heights. Use as wide a range of drop heights as possible. For each ball record the data in a table like the following:

| Elasticity data for large ball (d=  cm) |
|-----------------|-----------------|-----------------|
| $h_0$ Initial height (cm) | $h_1$ Rebound height (cm) | Elasticity |
| $\Delta h_0 = \pm \_\_$ | $\Delta h_1 = \pm \_\_$ | $h_1/h_0$ |

Record the diameter of each ball using the vernier calipers in case the diameter of the ball relates to any systematic error you discover when analysing the data.

Calculate the elasticity for each trial. Do the values seem reasonable? If not discuss your procedure and recording techniques with your partner to see if you can find the cause of the discrepancy.

The easiest way to calculate the mean and standard deviation of the mean (also called standard error) of the elasticity values is to use Excel. You can do this at home. The mean provides the Best Estimate of elasticity and the standard error is the uncertainty associated with that estimate.

NOTE: You will note that you don't actually use the errors $\Delta h_0$ and $\Delta h_1$ in the calculation of the standard error for your mean. Despite this, it is worthwhile to record them when you do the experiment in case you need them for comparison or additional calculations.
Study 3, Graphical Error Analysis

The trick to doing a graphical error analysis is to graph the data in such a way that a straight line is produced whose slope is related to the quantity of interest. In the super ball experiment this is straightforward. Rearranging equation (1) and solving for \( h_1 \) yields,

\[
h_1 = \text{Elasticity} \times h_0 \quad (2)
\]

Since the equation for a straight line is \( y = mx + b \) \((m=\text{slope}, \ b=\text{intercept})\) this means that the Elasticity will be the slope of a graph of \( h_1 \) versus \( h_0 \). It also implies that we expect the intercept \( b \) to equal zero in this case.

The advantage of a graphical analysis is it can be used to both qualitatively assess the data and to quantitatively analyze it to produce a numerical result with an error. Qualitative and quantitative assessment are discussed below.

Qualitative analysis using graphs

A major advantage of graphical analysis is that it helps you qualitatively analyze (or assess) your data. Consider the following graphs:

1. Does a line fail to pass through the origin, even though the theory predicts it should? This is a good indication of a systematic error that has shifted all the points on the line up or down. A strength of graphical error analysis is that it relies on the slope alone. It is therefore immune to additive systematic errors since these only shift the line vertically or horizontally but don’t change the slope. You should try to identify the source of the error before you leave the lab. If you can explain the cause it adds credibility to your experiment even though it may not change your results.

2. Do one or two points lie far from the line? If so, you should repeat these measurements to determine if a mistake was made or if the outlier actually represents a physical phenomenon that you did not anticipate. Obviously, this check can only be made if the data is graphed before you disassemble the apparatus so do a rough graph of the data before you leave! (The error bars give a sense of scale and can be used to determine if a point is “far” from the line.)

3. Are there systematic departures from the straight line? These can arise from a calibration error in one of your instruments, or because the model you have chosen for the fitting line is incorrect.
Quantitative Analysis using graphs

A linear regression of the \((h_0, h_1)\) data will produce a best fit model of the data in the form \(y = mx + b\). The slope, \(m\), will be our Best Estimate of Elasticity and the standard error on the slope is the uncertainty in this estimate \(^*\).

A plot of the regression line through the original points with error bars indicates whether the original error estimates are sensible. The best fit line should pass through the majority of the error ellipses (ellipses are defined by the x and y error bars). Note the following:

It should not pass perfectly through the middle of all of them - this indicates that the error estimates were exaggerated.

If it only cuts a few (or none) of the ellipses it indicates that the error was underestimated.

A plot of the residuals should be made (you can select this in Excel regression analysis) to more closely examine if there are systematic departures from a straight line. The residuals are the differences between the y values of the data points and the y values predicted by the linear regression equation. Ideally if a straight line is appropriate there should be no obvious pattern to the residuals.

**Procedure**

Use the sequence of \((h_0, h_1)\) points collected in experiment 2 to produce a large graph in your lab book (it should come as close to filling the page as choosing a workable scale allows). Don't forget to put in the error bars! Use this graph to do a Qualitative Analysis according to the outline given previously. Do any checks that your analysis indicates. Discuss your findings in your lab book. (Any graphs done in the lab in order to qualitatively assess the data should be divided up between members of the group in order to make the best use of your time. Work as a team to resolve any problems with the data.)

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\(^*\) This corresponds to a 67% confidence interval and is the same as using the standard error in the statistical analysis. This is what is generally used in physics. In other courses you may be required to use '2x standard error' as the uncertainty – this corresponds to a 95% confidence interval. Both methods are correct as long as it is clear which one you are using.
Worksheet: To be handed in

1. **Complete Experiments 2 & 3**

   Put your data from Experiment 2 into Excel and calculate the mean and standard error of the elasticity values [see online notes for a set-by-step guide to using Excel: UsingExcel.pdf].

   Use the same data for the graphical analysis as follows: [UsingExcel.pdf]
   
   - plot it (with error bars),
   - add the “line of best fit” and its equation to the graph (make sure the equation is in terms of $h_0$ and $h_1$ rather than $x$ and $y$),
   - do a linear regression of the data to obtain the standard error on the slope and intercept,
   - using the information from the linear regressions determine the Elasticity of each ball (with uncertainty).

2. **Answer the questions below.**

   [NOTE: Be careful to answer these questions in a scientific way; in particular, with due regard for the uncertainty of your measurements. When discussing causes of discrepancy and error there is not necessarily a right answer, just likely believable ones, and unlikely silly ones. The following are guidelines for reporting results:]

   - State results as **Best Estimate ± Uncertainty** (e.g. $x ± Δx$).
     - $Δx$ – the uncertainty must be quoted to the correct number of significant figures (see Appendix 3). Uncertainty in the final value will be one significant figure in most cases since the uncertainty in the measurements are usually only estimated to one significant figure.
     - $x$ - the last significant figure in any stated answer should be of the same order of magnitude (in the same decimal position) as the uncertainty.
     - Are two values “equal”? – [Two values $x$ and $x'$ are equal within experimental uncertainty if the ranges $x ± Δx$ and $x' ± Δx'$ overlap.]

**Questions:**

1. What is the **Elasticity** of each of the balls you tested? Summarize your results in the form of a table. e.g.
Elasticity values

<table>
<thead>
<tr>
<th></th>
<th>Single trial</th>
<th>Statistical analysis</th>
<th>Graphical analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball 1, diameter = 2cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ball 2, diameter = 5cm</td>
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</tbody>
</table>

2. How does the uncertainty using a single trial compare to the uncertainty of an estimate based on multiple trials (i.e. statistical and graphical analyses)?

3. Does the Elasticity vary with height? Make a graph of the residuals to justify your answer.

4. Is the elasticity of the two balls you tested the same? Justify your answer based on the estimate of Elasticity and its uncertainty.

5. What possible sources of systematic error did you try to eliminate? How?
   Do your graphs indicate that you were successful in eliminating systematic error (e.g. was the intercept of your graph zero within uncertainty)?

6. What are some sources of random error in this experiment? List them in order of decreasing magnitude, i.e. from most to least significant.

7. Based on the graph of the original data with error bars and the regression line comment on the measurement error estimates.

8. What else did you discover? Consider questions such as:
   ♦ Does the measured elasticity depend on the material the ball strikes (e.g. wood table top or vinyl floor)?
   ♦ Does it matter which side or part of the ball lands on the ground?
   ♦ Did it matter where on the floor the ball strikes?